


Session 2

Exercise 1 : Exact solution of the 1D Ising model without an applied field

The Hamiltonian is

$$H = -J \sum_{i=1}^{N-1} s_i s_{i+1} \quad \text{with open boundary conditions.}$$

Show that the free energy is: $Z_N = [2 \cosh(\beta J)]^N$

Trick: use recurrence!

Exercise 2 : Free-energy approach to the mean-field

The free-energy of a system is

$$F = U - TS$$

$$\text{with } U = \langle E \rangle = \sum_{\{\Gamma\}} E_{\Gamma} P_{\Gamma}$$

where $\{\Gamma\}$ is the set of all possible states for the system and P_{Γ} is the probability of a state Γ

$$S = -k_B \sum_{\{\Gamma\}} P_{\Gamma} \ln P_{\Gamma}$$

For the Ising model, use $P(\{s_i\}) = \prod_i P(s_i)$

and $m = (+1)P(+1) + (-1)P(-1)$.

Write the free-energy as a function of m and find the self-consistent equation.

